
The triangle-bisection illusion

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Abstract. In the triangle-bisection illusion, a dot is inscribed exactly halfway up the height of an equilateral triangle, but it looks apparently far more than halfway up. The illusion is found for second-order triangles defined by stereo depth and by equiluminous texture. It is strongest for equilateral triangles, and even stronger for concave triangles with curved sides. We propose that the observers are probably responding to the centre of area or centre of gravity of the triangle rather than to its half-height.

1 Introduction

In the little-studied triangle-bisection illusion (Piaget and Pène 1955; Robinson 1972/1998; Seckel 2000, illusion 108), a dot is inscribed exactly halfway up the height of an equilateral triangle, but it looks apparently far more than halfway up—much closer to the apex than the base of the triangle. In this paper we examine various properties of this illusion and show that it is strongest for equilateral triangles with angles of 60°. It still occurs for triangles that are defined by stereo depth (Julesz 1971) or by equiluminous textures, and this rules out any explanation based upon simple spatial filtering (Morgan 1996). It is isotropic, being equally strong along all three axes of an equilateral triangle, so there is nothing special about vertical. It is much increased when the sides of the triangle are curved inward to make a concave, bottom-heavy triangle, and reduced when the sides are convex and bulge outwards. It is stronger when the bisection spot is replaced by a grey diamond (see figure 7). We conclude that observers are responding, not to the half-height of the triangle as they were asked to do, but to the centroid of the triangle (Morgan and Glennerster 1991) as defined by its centre of area or centre of gravity.

2 Experiment 1: Measuring the effect

Observers were undergraduate students who received course credits for their participation. They viewed various forms of equilateral triangles on a computer monitor screen. They adjusted the vertical position of a small spot inside the triangle, until it appeared to be halfway up the triangle. Results were recorded.

2.1 Method

The programs we used were written in Macromedia Director 10.1 on a Macintosh computer running Mac OS 10.4. The screen was viewed from a distance of 57 cm in a dimly lit room. Five different background shapes were used: (i) an outline equilateral triangle; (ii) a Kanizsa triangle defined by three pacmen, each with a bit taken out of it; (iii) three small dots arranged in a triangle; (iv) a T-shaped region made by removing parts of a solid equilateral triangle; and (v) an outline pentagon.

The stimuli are shown in figure 1. The height of each triangle was 6 deg of visual angle. Each of these triangles or pentagons was presented upright on half the trials

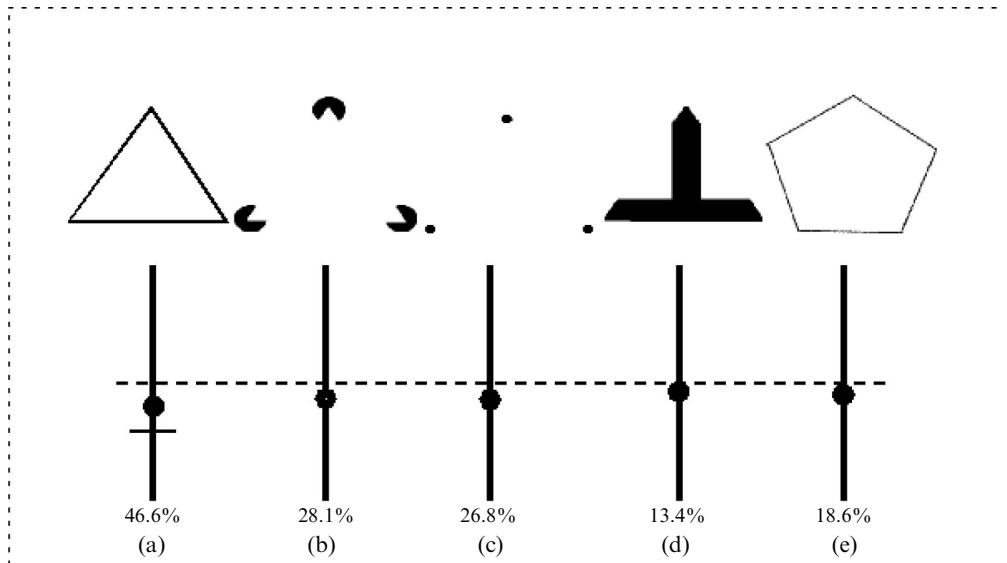


Figure 1. The top row shows various forms of equilateral triangle. Each vertical line represents the full height of the triangle, and the solid circle shows the mean position selected as half-way up the height (mean of three observers \times 15 trials). The dashed horizontal line shows the true halfway position. Numbers show the percentage size of the illusion, $100 \times [(\text{upper segment}) / (\text{lower segment}) - 1]$.

and upside down on the other half, to reduce the effects of any top bias or bottom bias. The five shapes and two orientations (upright or upside down) were presented in random order. Fifteen settings were made in each condition by three observers (PH, SA, and one naive observer). The observer could strike one of two keys, one to move the dot up and the other to move it down, until it appeared to be exactly halfway up the triangle (or pentagon). The observer then hit the space bar to record the setting, which was recorded for later analysis offline.

2.2 Results

As one would expect, a dot halfway up that looked ‘too high’ in an upright triangle consistently looked ‘too low’ in an upside-down triangle. Accordingly, for the analysis we inverted the results for upside-down shapes and pooled them with the results for upright shapes.

The settings obtained (mean of three observers \times 15 trials) are shown in figure 1. Under each of the five stimuli is drawn a vertical line representing the height of the triangle (or pentagon). The dashed horizontal line is actually halfway up the line, but the black spot on each line shows the observers’ mean settings, which generally lay below the true halfway point. The downward displacement of each spot below the halfway point represents the triangle-bisection illusion.

In all cases, the true setting looked ‘too high’ and all observers set the position of the triangle bisector below its correct position, so that the upper segment was physically longer than the lower. To calculate the size of these illusions, assume that the triangle is 200 pixels high, so the veridical bisector position is at 100 pixels, but that the observer selects a setting s lower than this ($s < 100$ pixels). We express the percentage illusion as $100 \times [(\text{upper segment}) / (\text{lower segment}) - 1]$. Thus a veridical setting, where $s = 100$ pixels, would yield zero illusion; and a setting s to (say) 90 pixels would give $100 \times [110/90 - 1] = 22.2\%$; so in this example the two segments would look perceptually the same when the upper segment was actually 22.2% longer than the lower segment.

The percentage results obtained are shown below each stimulus. Thus, a hypothetical spot set halfway up an outline equilateral triangle (figure 1a) and lying on the dashed horizontal line looked 'too high' and, to make it look subjectively halfway up, it was set below the halfway point, with the upper segment 46.6% longer than the lower. The illusion was also strong for a Kanizsa triangle (28.1%) and for three dots arranged in a triangle (26.8%). However, the T-shape and the pentagon gave smaller illusions (13.4% and 18.6%, respectively). The T-shape in figure 1d is a compromise between the triangle bisection illusion and the bisected-T illusion (reviewed by Robinson 1972/1998).

3 Experiment 2: Aspect ratio of the triangle

We examined the effect of the aspect ratio of the triangle upon the illusion. Seven solid black triangles were presented in random order against a white surround. All triangles had equal areas, but their aspect ratios (height/width) formed an equally spaced logarithmic series, namely 1 : 4, 1 : 2.52, 1 : 1.58, 1 : 1, 1.58 : 1, 2.52 : 1, and 4 : 1. This meant that the apical angles were 132°, 111°, 85°, 60°, 40°, 27°, and 17°. Miniature versions are displayed as icons along the x-axis of figure 2. The widest triangle was 9.5 deg wide and 2.4 deg high, while the tallest triangle was 2.4 deg wide and 9.5 deg high. The seven triangles were presented eight times each in random order, and they were randomly made upright or upside down (base-down or base-up) on each trial. The initial vertical positions of the triangle and of the adjustable spot were also randomised on each trial.

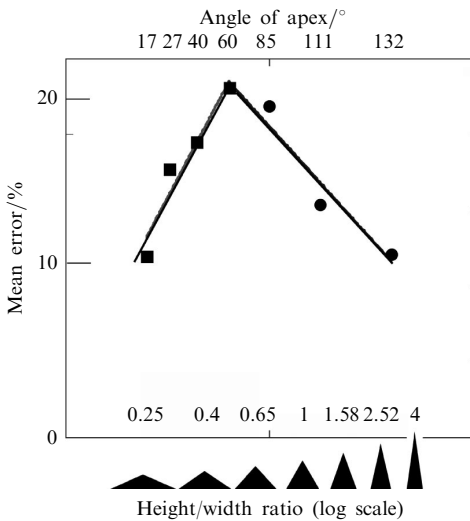


Figure 2. Bisection illusion for equilateral triangles of different aspect ratios.

The observer adjusted a small spot until it apparently lay halfway up the triangle, and pressed the space bar after each setting. All settings were recorded for later analysis offline.

3.1 Results

Figure 2 (mean of three observers \times 8 readings) shows that the triangle-bisection illusion was strongest for an equilateral triangle with an apical angle of 60°. We do not know why this is so, but it is consistent with Piaget and Pène (1955), who found a maximum effect at an angle of 55°. However, this angle was not really critical, and the bisection illusion was found over the whole range of aspect ratios tested.

4 Experiment 3: Cyclopean triangle

The experiment was repeated with the same procedure but with a cyclopean triangle (Julesz 1971). Each eye saw a random-dot field, presented side-by-side on the monitor screen and fused binocularly by means of a mirror stereoscope. A triangular area of random dots was presented in crossed disparity. When one eye was closed, no triangle was visible, because the triangle did not exist within one eye's view, but was present only as a correlation between the eyes. But when the stimulus was viewed stereoscopically, the observer saw an equilateral triangle, 5 deg in height, floating in depth in front of the background. A small red spot could be moved up and down the mid-line of the triangle, as before, and the observer's task was again to set it apparently halfway up the triangle. Results (mean of two observers \times 10 readings): observers set the upper segment to be 38.1% longer than the lower.

This shows that the illusion does not require that the triangle be a first-order shape defined by luminance—instead, a second-order textured shape defined by binocular disparity suffices. It also shows that the neural site of at least some components of the triangle illusion, as for some other geometrical illusions (Coren and Porac 1984), comes after the point of binocular fusion.

5 Experiment 4: Texture-defined triangles rule out simple spatial filtering

Some geometrical illusions, such as the Zöllner and Fraser stimuli, can involve luminance-filtering operations (Morgan and Casco 1990; Morgan et al 1990, 1995; Morgan 1996). Thus, if the Münsterberg and Café Wall illusions are filtered by difference-of-Gaussian filters that resemble the receptive fields of retinal ganglion cells, the output contains a series of tilted lines, alternately black and white, instead of the long horizontal line in the original stimulus (Morgan and Moulden 1986). These tilts have the same orientation as the perceived illusions. Could the triangle-bisection illusion be the result of such spatial filtering? Experiment 4 suggests that the answer is 'no', because we were still able to obtain a bisection illusion from second-order triangles that were defined by equiluminant textures. Since these triangles had the same mean luminance as the surround, any luminance filtering operations would be blind to these triangles.

We used two pairs of textures to define the equilateral triangles, which all had a side of 6 deg. One pair consisted of left-oblique versus right-oblique hatchings. The other pair consisted of a grey halftone dot texture versus a cross-hatching of tiny diamonds made by superimposing left-oblique and right-oblique hatchings. Examples are shown in figure 3. (These are not exact replicas; the actual hatchings were much finer than the reproductions in figure 3.) On half the trials, the first texture of a pair filled the triangle and the second texture filled the background. On the other half of trials the reverse was the case. As a control condition we used white triangles with a black surround. (We did not use black triangles on white because this would have hidden the black cursor.) Each triangle was upright on half the trials and upside down on the other half. Textures and triangle orientation were randomly selected on different trials. Three observers made 6 trials in each of the ten conditions (2 orientations \times 4 textures + black/white), placing the cursor onscreen at the perceived halfway point and clicking the mouse to record their settings, using the same procedure as in experiment 3.

Results (mean of three observers \times 6 trials) are shown in figure 3: the black–white triangle and the mean of the texture-defined triangles showed almost identical illusions, in which observers set the upper segment of the subjective 'bisection' to be respectively 1.47 times and 1.5 times the length of the lower segment. Thus, in both cases they set the apparent bisection point three-fifths instead of one-half of the way down the height of the triangle. Taken together, experiments 3 and 4 lead us to conclude that second-order triangles, defined either by stereo disparity or by texture, gave the same illusion

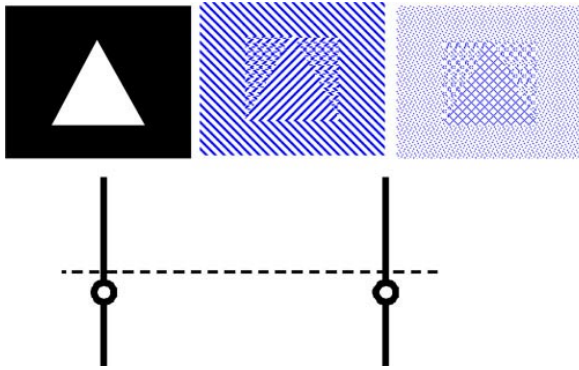


Figure 3. [In colour online, see <http://dx.doi.org/10.1068/p5866>] Texture-defined triangles (right) gave the same bisection illusion, shown below as circles on the black vertical lines, as luminance-defined triangles (left).

as first-order, luminance-defined triangles. Since spatial filtering would be sensitive to luminance edges, but blind to edges of disparity or texture, we conclude that simple luminance filtering is not involved in the bisection illusion. Admittedly, there might still be a texture grabber (Werkhoven et al 1993) followed by some form of spatial filtering, but this would be a higher-level filter, not a low-level luminance filter; and we know of no independent evidence for the role of such hypothetical filters in geometrical illusions.

6 Experiment 5: Three-way bisections

So far, all subjective halfway points selected lay along the vertical axis that bisects a triangle. However, in experiment 5 we measured halfway points along all three axes of an equilateral triangle. Each of these axes bisects an angle and its opposite side, and the axes are oriented at the vertical, and 60° to left and right of vertical.

The stimulus was an outline equilateral triangle of 7 deg side. Initially all three sides were dark blue. On each trial, however, a randomly selected side was made dark red, and this signaled to the observer that the task was first to move the mouse cursor (a small cross) to the corner opposite that edge, then move the cursor freely back and forth between the corner and the red edge until a point was found where the cursor appeared to bisect the distance between edge and corner. The observer then clicked the mouse to indicate this subjective halfway point, and a new side was randomly selected for the next trial. Six naive observers made five settings for each side.

Results are shown in figure 4. The small cross shows the centre of gravity, which is also the centre of area. It is also the centre of area, since any straight line passing through this point would divide the triangle into two

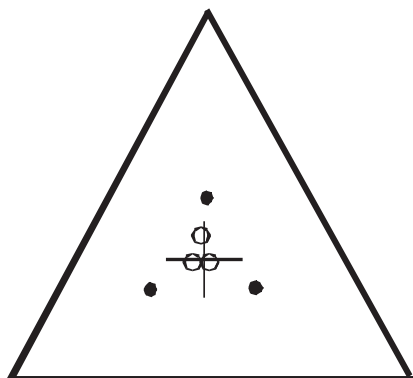


Figure 4. Results of experiment 5. The cross shows the centre of gravity, which is also the centre of area. Each filled circle shows the halfway point between the nearest corner and its opposite side. Open circles show the observers' settings of the apparent halfway points along the triangle's three axes.

regions of equal area. It is also where the bisectors of the three angles of the triangle intersect, and it lies two-thirds of the way from a corner to its opposite side.

The three filled circles show the objective halfway points, whilst the open circles indicate the settings of the subjective halfway points selected by the observers. Each objective halfway point gives the illusion of looking subjectively ‘too near’ to its corresponding corner, whilst the subjective halfway points cluster closely around the centre of gravity of the triangle. Results clearly support the notion that observers are judging the centre of gravity (or centre of area; in fact, for an equilateral triangle these two are the same).

The results also show that the illusion is isotropic, with no special emphasis upon the vertical. Note that the term ‘centre of gravity’ has no implications about the vertical. The only difference is that for a centre of area all pixels in a shape are of equal value, whereas to compute the centre of gravity each pixel is multiplied by its distance from the fulcrum.

7 Experiment 6: Trapezia

We started with an equilateral triangle with a side of 12 cm. We then made a series of trapezia of different widths, by splitting the triangle down its vertical midline and inserting rectangles of various widths, namely 0, 2, 4, 6, 8, and 10 cm. Thus the top sides of the trapezia were 2, 4, 6, 8, and 10 cm, while the bottom sides were 14, 16, 18, 20, and 22 cm. Observers were asked to bisect each trapezium by setting the cursor to look halfway up its vertical midline, using the same procedure as in the previous experiments. Our prediction, which we confirmed, was that the wider the trapezium, the more it would dilute the bisection illusion.

Results are shown in figure 5. In this figure, the settings (mean of five naive observers \times 10 trials) are shown as open circles, and veridical settings would be at $y = 50\%$. Values of y between 40% and 50% show that the observer set the cursor ‘too low’, demonstrating the standard bisection illusion. The crosses in the figure show the calculated centres of area of each trapezium. In the limiting cases, the centre of area lies 33% of the way up an equilateral triangle and 50% of the way up a rectangle.

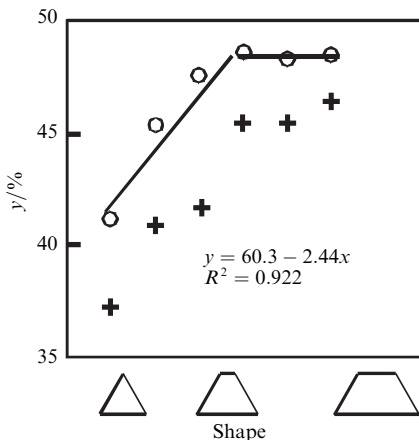


Figure 5. Halfway judgments for triangles and trapezia (mean of 5 subjects \times 10 trials). Subjective halfway points (open circles) were a compromise between the actual halfway points (top edge of graph, $y = 50\%$) and the centres of area (crosses) of the shapes.

Figure 5 shows that the strongest illusion ($y \simeq 40\%$) occurred for an equilateral triangle, with the illusion declining as the trapezium was made wider and saturating when the width of the added rectangle was 6 cm or more. It also shows that for every shape the observers’ mean settings were displayed toward the centre of area.

8 Experiment 7: Triangles with curved sides

Experiment 7 was similar to experiment 1 except that the triangles were now distorted. We reasoned that the bisection illusion should be reduced if the sides of the triangle bulged outwards to make it closer to a circular disk, and it would be exaggerated if the sides of the triangle were sucked inwards, making it more ‘bottom-heavy’. These predictions are compatible with the centre-of-gravity theory proposed later in section 13.

Accordingly, we made seven new triangular shapes whose sides were composed of straight lines, circular arcs, or a mixture of both. The radius of the circular arcs was equal to the side of the equilateral triangle. Each was presented both upright and upside down, eight times in random order. Results for upright and upside-down figures were pooled for analysis. The stimuli we used are shown in figure 6.

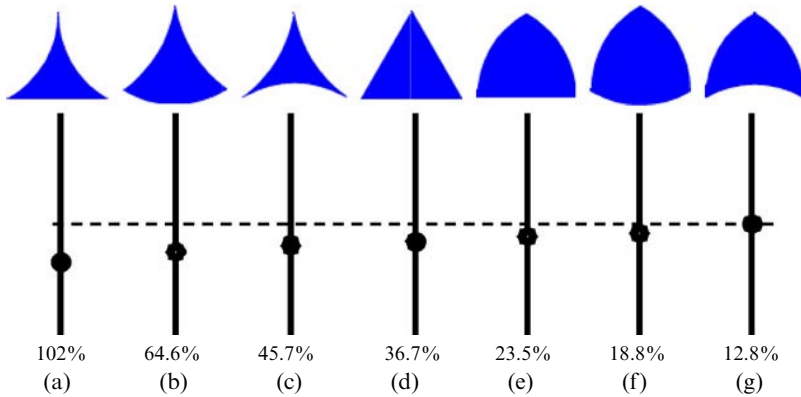


Figure 6. [In colour online.] Bisection illusion for triangles with curved sides was greatest for triangles with concave sides [(a), (b), (c)] and smallest for triangles with convex sides [(e), (f), (g)].

8.1 Results

The percentage illusions, calculated as before, are shown underneath each stimulus in figure 6 (mean of three observers \times 16 readings). The strongest illusion, in which the upper segment was set almost exactly twice as long as the lower segment, was obtained with a triangle with concave sides and a flat base. Note how strong this illusion is: it is hard to believe that the spot on the vertical line in figure 6a was actually perceived as halfway up the line when it was embedded in the concave triangle. In fact, all triangles with concave sides gave large illusions; whilst triangles with three straight sides gave a smaller illusion, of 36.7%, not too different from the 46.6% obtained in experiment 1; and triangles with convex sides gave the smallest illusions.

9 Experiment 8: Bisecting a triangle with a diamond

We now repeated experiment 1a, replacing the adjustable spot with an adjustable grey diamond. A white equilateral triangle of 6 deg in height, set in a black surround, was exposed on the monitor screen. A grey diamond filled the upper part of the upright triangle (figure 7). The observer’s task was to hit keys that moved the bottom of the diamond up and down, until its bottom tip appeared to pass through the centre of the upright triangle. (The top of the diamond always fitted snugly into the top of the triangle.) To minimise top–bottom bias, the entire figure was randomly made upright or upside down (base-down or base-up) on each trial.

Results for the upright and upside-down triangles were pooled and averaged. Figure 7 shows the mean setting of the diamond, together with a spot that actually lies halfway up the triangle. If the observers had been veridical then the diamond’s tip would have passed through this central point, and the upper segment defined as the

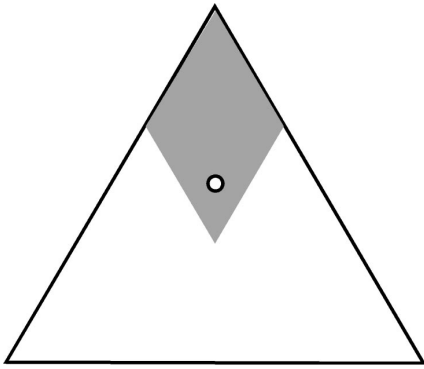


Figure 7. Observers attempted to set the bottom tip of the diamond to be halfway up the triangle. Small circle shows the actual halfway point.

vertical height of the grey diamond would have been equal to the lower segment, defined as the gap between the bottom of the diamond and the bottom of the triangle. In fact, however, they set the diamond much lower than this, with the upper segment 81% longer than the lower segment (mean of five observers \times 10 trials).

10 Experiment 9: Opposed triangles with curved sides

We now combined the curved sides of experiment 7a with the diamond arrangement of experiment 8, and this gave the largest illusion of all. Figure 8 shows three versions of this new display. In all three versions, the height of the diamond is EF and the height of the triangle is EG. At the correct bisection point, which is shown in figures 6a, 6b, and 6c, the distances $DE = EF = FG$, so that EF is one-third the length of DG. However, a moment's inspection shows that in each case the diamond EF looks far too short.

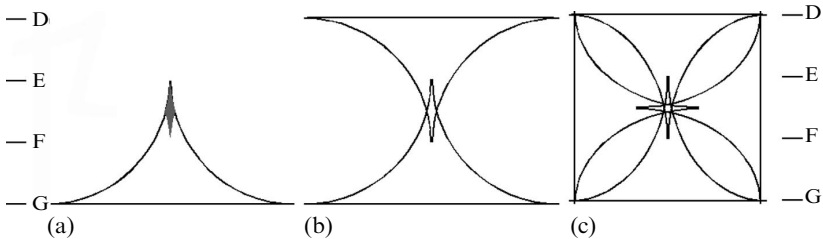


Figure 8. Illusions of bisection in concave triangles: (a) the bottom of the grey concave diamond lies halfway down the triangle; (b) the apex of each triangle lies halfway down the other triangle (a version of this illusion was measured in experiment 6); (c) the length of each diamond in the star is equal to the gap between the diamond's tip and the side of the large square. In each case, $DE = EF = FG$, but EF looks much shorter than FG.

We measured the illusion of figure 8b, in which two triangles with curved sides are superimposed, one being upside down. The observer struck two keys that moved the triangles vertically in opposite directions, either closer together or further apart. The task was to adjust the vertical separation between the two triangles until the apex of each triangle lay at the halfway point of the other triangle. (The symmetrical stimulus ensures that when the apex of the upper triangle bisects the lower triangle, then the apex of the lower triangle also bisects the upper triangle.) In figure 8b, the triangles do bisect each other and $DE = EF = FG$. However, to attain a subjective bisection the observers made the triangles overlap much more, until the height of the diamond EF was more than twice as long as DE and FG, with an illusion (mean of nine observers \times 5 trials) of 114%.

11 Experiment 10: Common fate and attention

The triangle-bisection illusion affects the perceived position of a spot within a triangle. We now asked whether this shift was local to the triangle, altering only the criterion of what is halfway up; or whether the shift was more global and applied to the position of the whole triangle.

We superimposed two semitransparent outline triangles, an upright (base-down) red triangle and an upside-down (base-up) blue triangle. The apex of each triangle lay on the mid-point of the base of the other triangle (as in figure 9a). A central spot lay halfway up one triangle, and therefore halfway down the other. Thus, if the observer was able to attend to one triangle and ignore the other, the spot should appear to be shifted apparently up, or down. We used ‘common fate’ to control attention. Each triangle jittered horizontally at random and independently, so that they shifted irregularly back and forth across each other. The spot was at first red and synchronised with the red triangle, moving horizontally in step with it. It then abruptly became blue and was synchronised with the blue triangle; and this cycle continued indefinitely. Thus the spot followed the motion regime first of one triangle, then of the other.

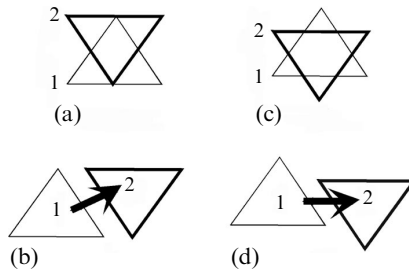


Figure 9. (a) Two equilateral triangles were superimposed with their upper and lower boundaries lined up. This also aligned their halfway points. When flashed in alternation, motion was seen upward from time 1 to time 2 (not shown); (b) when alternated side-by-side, motion was seen obliquely upwards. (c) When centres of gravity coincided, in a Star of David configuration, no net vertical motion was seen (not shown); (d) when then alternated side-by-side, the apparent motion was horizontal with no vertical component. *Note:* For clarity, triangles at time 2 are drawn with heavy lines. This was not so in the actual experiment.

11.1 Results

The qualitative result was that when the moving spot synchronised with the base-down triangle, it appeared to be lower than halfway down, and when it synchronised with the base-up triangle it appeared to be higher than halfway down—relative to the triangle that moved with it. It did not appear to move vertically with respect to the page or to the observer. So when the spot was perceptually grouped with one triangle or the other, its position was judged relative to that triangle.

12 Experiment 11: Does the illusion shift the spot, or the triangle?

Since the spot in experiment 10 appeared to change its vertical position relative to the triangles, but not its absolute vertical position, we argued that perhaps the illusion shifted the perceived position not of the spot but of the triangles. We measured this directly by superimposing two triangles, as in experiment 10, with the apex of each triangle on the midpoint of the base of the other triangle (figure 9a). There was no spot. In an apparent-motion design, we briefly flashed up first the upright, then the upside-down triangle, and asked observers to report whether they saw any net motion up or down. All observers reported net motion upwards. They were then invited to adjust the position of the base-up triangle (downwards), until they saw no net motion upwards or downwards. They then struck the space bar to record their setting, which was recorded for later analysis offline. Each observer made five settings.

12.1 Results

The two triangles gave minimum net vertical motion, not when superimposed as in figure 9a, in other words when their halfway points coincided, but instead when their centres of gravity coincided, so that, if superimposed and stationary, they would form a Star of David. Clearly the visual system assesses the position of the motion tokens, not by their halfway points, but by their centres of gravity. This result for apparent motion is consistent with the centre-of-gravity account of the triangle-bisection illusion.

13 Discussion

Let us summarise our results. We found a strong triangle-bisection illusion, in which the distance above the subjective halfway point was 46.6% or 36.7% greater than the distance below it (experiment 1, figure 1a; and experiment 7, figure 6d). An equilateral triangle gave the best illusion (experiment 2, figure 2), and a cyclopean triangle also gave the effect, showing that the illusion occurs after the anatomical point of binocular fusion. We found a novel, extremely strong version of the illusion (102%) when the triangle had two concave sides, emphasising the triangle's 'bottom-heavy' geometry. Also an equilateral triangle gave a very strong illusion (81%) when the adjustable spot was replaced by an adjustable grey diamond. Two opposed triangles with curved, concave sides gave the biggest illusion (114%) in experiment 9. These findings suggest that the observers were tending to equalise the areas above and below the subjective halfway point.

We propose two theories to explain the triangle-bisection illusion: (1) inappropriate constancy scaling; and (2) centre of gravity or area. These theories might compete or overlap.

13.1 *Inappropriate constancy scaling*

Gregory (1968) has suggested that many geometrical illusions contain perspective cues that trigger an inappropriate constancy scaling, which effectively increases the perceived size of more distant objects. Thus an equilateral triangle might be perceived as the flat picture of a road seen in perspective, with the top apex lying at infinity and the bottom base as the nearest part of the road.

However, we do not believe that constancy scaling operates in the triangle-bisection illusion because the perceived effects are in the wrong direction. Suppose, then, that the triangle were a perspective photograph of a road, with the apex of the triangle representing the road's width dwindling to zero at the horizon. A spot halfway up the triangle would bisect the photograph up-down, but it would not bisect the road near-far. The region above the spot would correspond to a very long stretch of distant road, whilst the region below the spot would represent a very short stretch of nearby road. Perceptual size constancy would expand the (distant) region above the spot, so the spot would look more than halfway down the triangle. In fact, however, it looks less than halfway down (Humphrey and Morgan 1965). So constancy scaling predicts the wrong direction for the triangle-bisection illusion.

13.1.1 *Ponzo illusion.* The midline of the triangle in figure 10a has a gap exactly halfway down, but this gap looks 'too high'—in other words, the upper line segment looks apparently shorter than the lower. A simple rotation (b) and expansion (c) turn this into the familiar Ponzo illusion (Ponzo 1928), in which the upper line looks longer. The Ponzo illusion has been attributed to inappropriate constancy scaling (Gregory 1968), but it goes in the opposite direction to the triangle-bisection illusion.

In fact, we do not believe that Gregory's theory is applicable to the triangle-bisection illusion, and we certainly do not regard this illusion as a test of his theory. Note that the grey diamond in figure 7 greatly enhances the illusion, but there is no plausible perspective interpretation of the diamond. And the triangle with concave sides in figure 4a

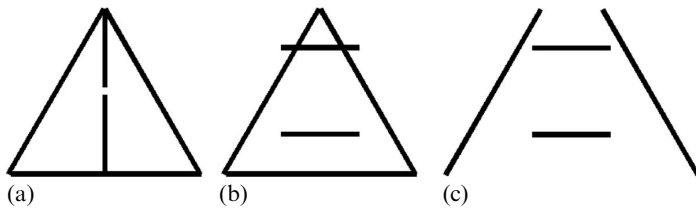


Figure 10. (a) The triangle-bisection illusion. The gap in the midline is halfway up but looks ‘too high’, so the upper line segment looks shorter than the lower. (b) Each segment has been rotated about its own centre through a right angle. Now the upper segment looks longer. (c) Moving the oblique sides of the triangle further apart reveals the familiar Ponzio illusion, in which the upper line also looks longer. If the illusions in (b) and (c) are caused by inappropriate constancy scaling, then the triangle-bisection illusion in (a) cannot be.

gives a huge illusion, but any perspective theory would predict the opposite, since the concave triangle would represent (if anything) a road that comes down a distant hill and flattens out nearer toward the observer. On any perspective story, the steeper hill in the distance would represent a smaller amount of physical road than a flat horizontal road in the distance, so a perspective theory would predict a smaller bisection effect for a concave triangle, not the larger effect found here.

13.2 Centre of gravity/centre of area

It may be that, although observers are asked to find the halfway point, which has equal linear extents above and below it, they are actually finding the centre of area, which has equal areas above and below it. They are applying a 2-D algorithm to a 1-D problem. The centre of area of an equilateral triangle lies well below its halfway point, and there is evidence that observers make settings that are a compromise between the two. In figure 1a, the long dashed horizontal line passes through the half-height, and the short horizontal line just below it passes through the centre of area (= centre of gravity).


For triangles, although not for geometrical shapes in general, the centres of gravity and of area coincide. At the least, we think it unlikely that observers simply misunderstood the instructions, so they were not doing the wrong task by misguidedly aiming for the centre of area or mass instead of for the halfway point. A glance at figure 7 shows that the small circle that marks the physical halfway point looks subjectively far above halfway, whilst the tip of the grey diamond looks about right for most observers, even though the distance above the grey tip is 81% longer than the distance below it. In other words the illusion is genuinely visual and not merely a linguistic mistake.

What is true for triangles is also true for clusters of dots. There is evidence that observers have a strong tendency to locate a cluster of dots at its centroid, even if asked to locate it at a particular dot differing in colour from all the rest (Harris and Morgan 1993; Morgan et al 1990). These authors argue that observers have an automatic strategy of locating centroids because that is what the visual system is set up to do. Observers are about as good at locating the centroid of a dot cluster as they are at locating a single dot. This is not done by a blurring strategy, since thresholds are the same when all the dots of the cluster have the same polarity as when half are black and half are white on a grey background (Morgan and Glennerster 1991). We conclude that our observers were unable to abstract the halfway point of a triangle, as they were asked to do, but instead were locating the centroid of the triangles.

In sum, we conclude that observers are setting the spot, not to the halfway point that has equal linear extents above and below it, but to somewhere near the centre of area, which has equal areas above and below it. This is because the purpose of the visual system is to locate objects and their centroids, rather than to locate abstract measures such as the halfway point of objects.

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